Large Q factor with small ring cavities

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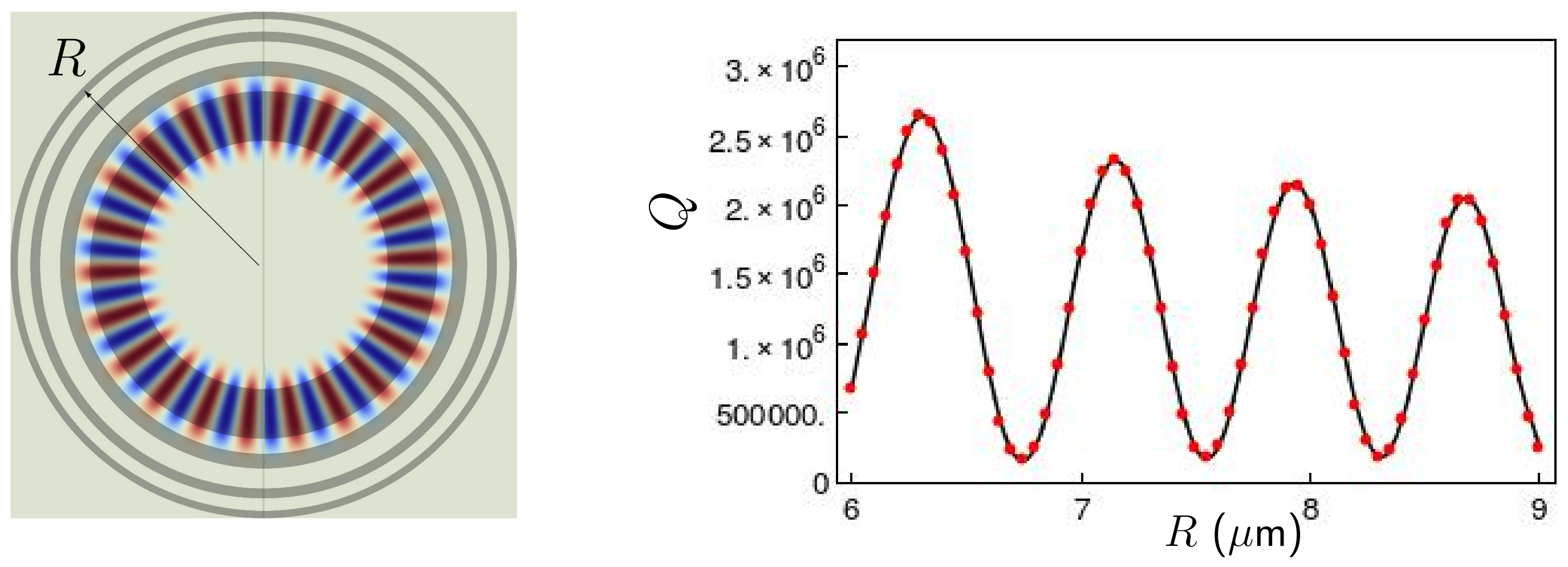
Introduction

Mirco-ring resonators and other Whispering-Gallery-Mode (WGM) resonators are key components for photonic application and more fundamental light-matter studies. One figure of merit as far as linear interaction are concerned (reactive sensing, lasing, quantum optics) is the Purcell factor $\mathcal{F} = 6\pi Q/(k^3V)$. It shows that what is generally desirable is a combination of large Q factor and small volume V. This is also true for nonlinear interaction. Limitation on Q are of two types:

- Material related: parasitic absorption or diffusion by impurities, surface roughness, . . . ,
- Fundamental: bending losses, which increase very rapidly as V diminishes

Here, we propose a new scheme to reduce bending losses to any desired level and reach material-limited Q, even for small V. We propose to enclose the cavity by a shield made of concentric rings to effectively produce radial Bragg reflection. In this way, the radiation by WGM can be quenched. We will find that the method is very effective. However, it cannot be understood as simply as by conventional Bragg reflection. Indeed, (1) the radial oscillation of the field are not with constant period, (2) the wave fronts can be strongly curved, and (3) if the Bragg shield is radially misplaced, then WGM radiation can be enhanced rather than quenched -something that does not happens in conventional Bragg reflection. Importantly, the field stay confined in the cavity and is not redistributed to larger radii.

Schematically, the situation is depicted below:



Left: Al$_2$O$_3$ ring cavity ($n = 1.65$) with radius 3.2μm surrounded by a 3-ring shield. In the absence of the shield, at a wavelength $\lambda = 1.26\mu$m, $Q_0 \approx 15000$ but the material-limited valued is $\sim 2\ 10^6$. Right: Q factor in the presence of the shield as a function of the geometrical parameter R. Here, 3 rings are sufficient to enhance Q by a factor $Q/Q_0 \approx 170$ and reach material-limited values.

Theory

In 2D, just outside the cavity, the field is generally given by

$$\psi = \left[a_0 J_\nu(kr) + b_0 Y_\nu(kr)\right] e^{i(\nu\theta - kct)}$$

while outside the shield, one has

$$\psi = \left[a J_\nu(kr) + ia Y_\nu(kr)\right] e^{i(\nu\theta - kct)},$$

where $k = k_r - ik_i$ is the complex wave number in vacuum and J_ν, Y_ν are the usual Bessel functions. Between the complex coefficients describing the field in the two region on can derive a matrix relation

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = S(k) \begin{pmatrix} a \\ ia \end{pmatrix}.$$

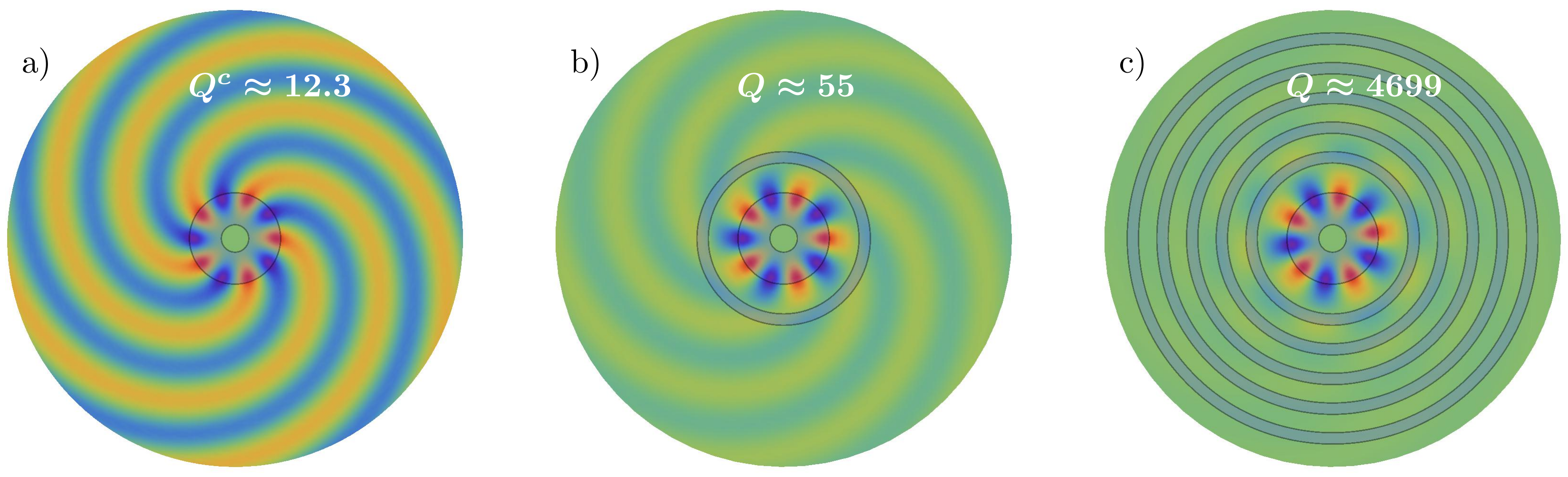
The matrix $S(k)$ characterises the shield and combines Bessel functions evaluated on either sides of each interface that make up the shield. The presence of the shield affects the values of k at which WGM exist. It thus also affect the imaginary part k_i, which measures the radiation losses. Finding the complex resonances of the composite structure cavity+shield is numerically complicated. However, we have shown [1] that, given the (real part of) the wave number of the bare cavity, k_r^c, one has the enhancement

$$\boxed{\frac{Q}{Q^c} \approx \left\{ \mathrm{Re} \left[\frac{S_{11}\left(k_r^c\right) + iS_{12}\left(k_r^c\right)}{S_{22}\left(k_r^c\right) - iS_{21}\left(k_r^c\right)} \right]. \right\}^{-1}}$$

This formula allows us to efficiently and rapidly optimise the geometrical parameter of the shield.

Below is a demonstration on an extreme example: a 1μm-radius Al$_2$O$_3$ cavity ($n = 1.65$) operating at $\lambda = 1.45\mu$m and which contain only 5 oscillations along its perimeter. Using 5 shells, an enhancement factor of 370 is reached.

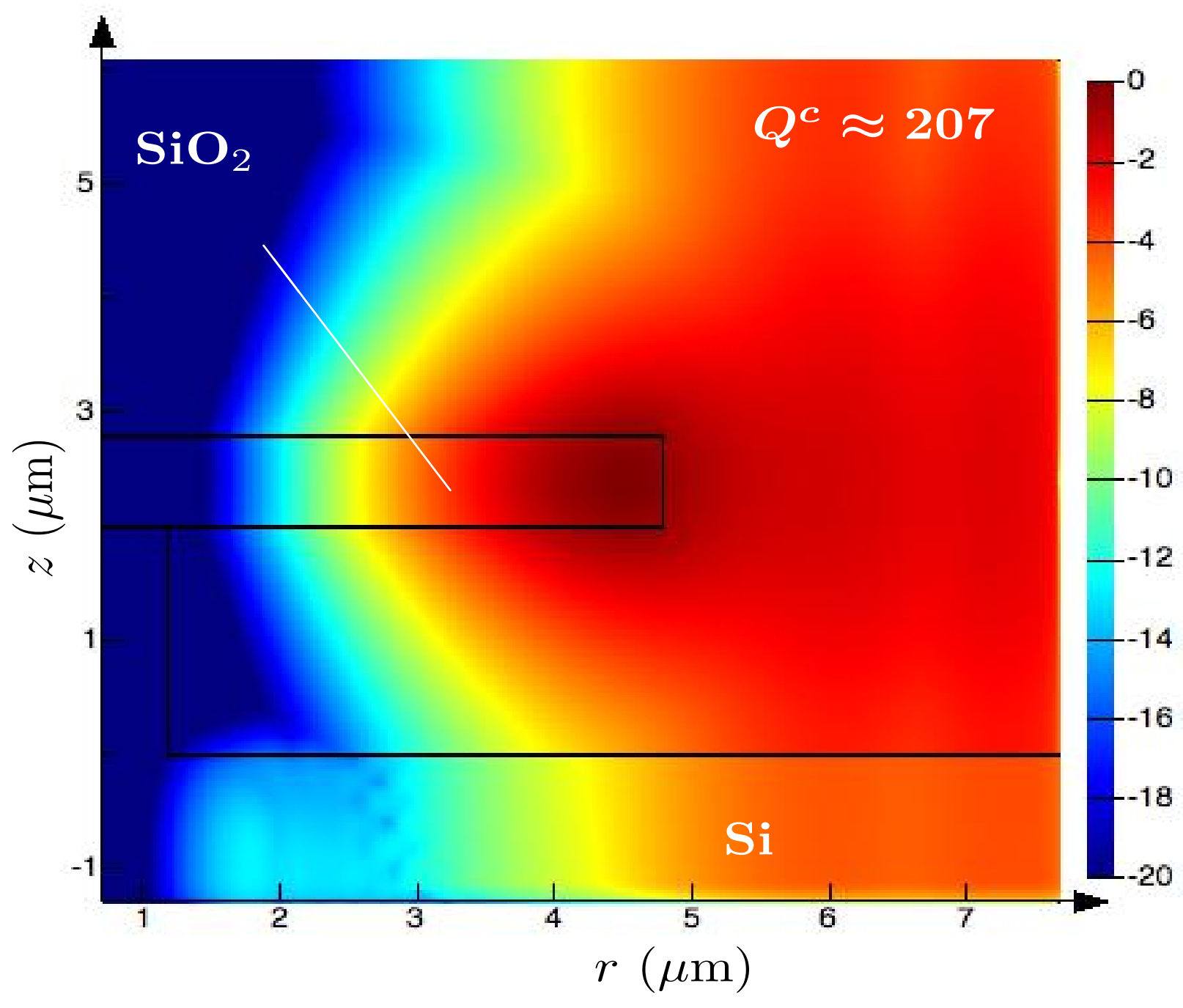
Note: Al$_2$O$_3$ is a promising material that can be doped with rare-earth material, such as Yb with lasing line around 1.26μm [2].

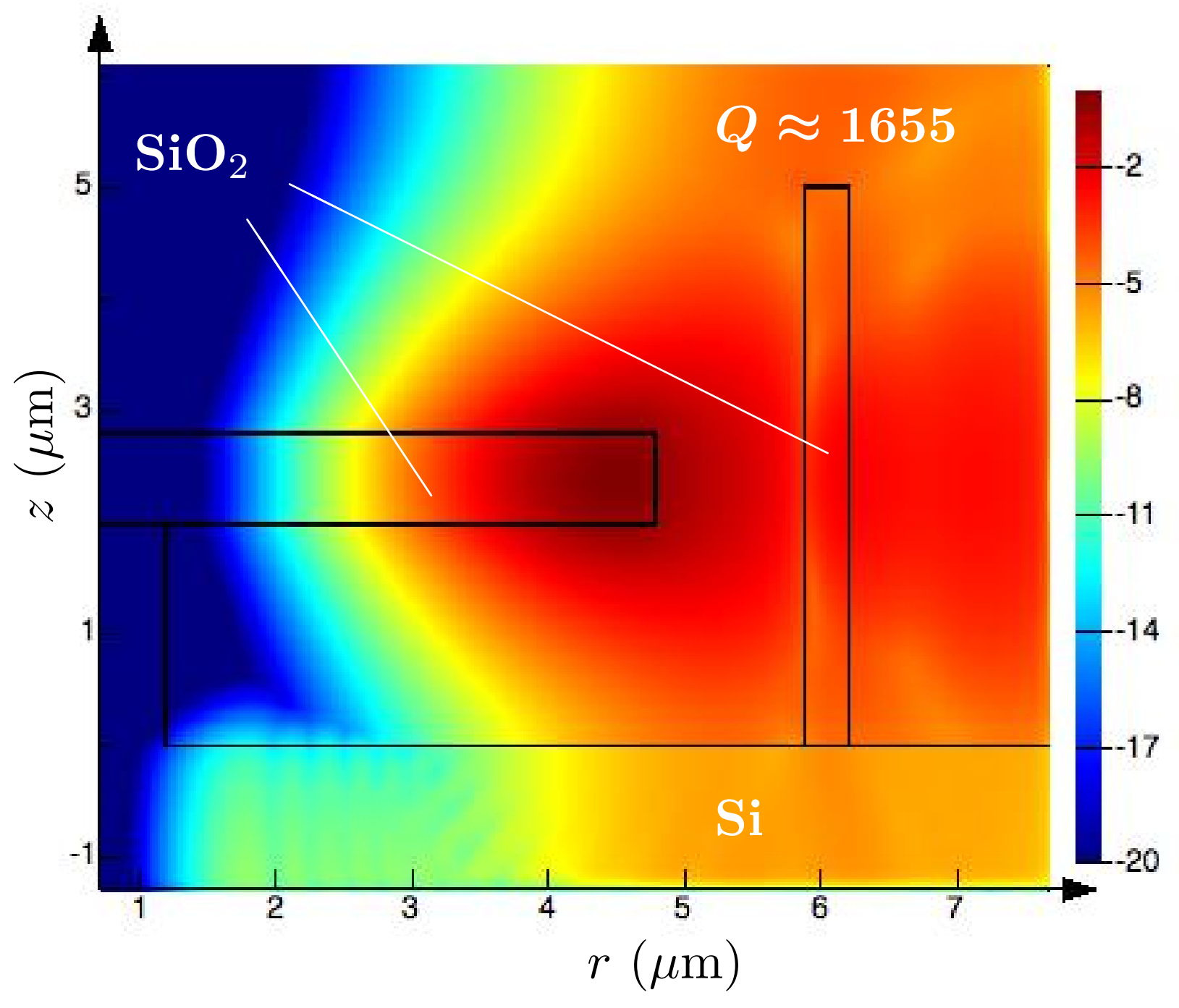


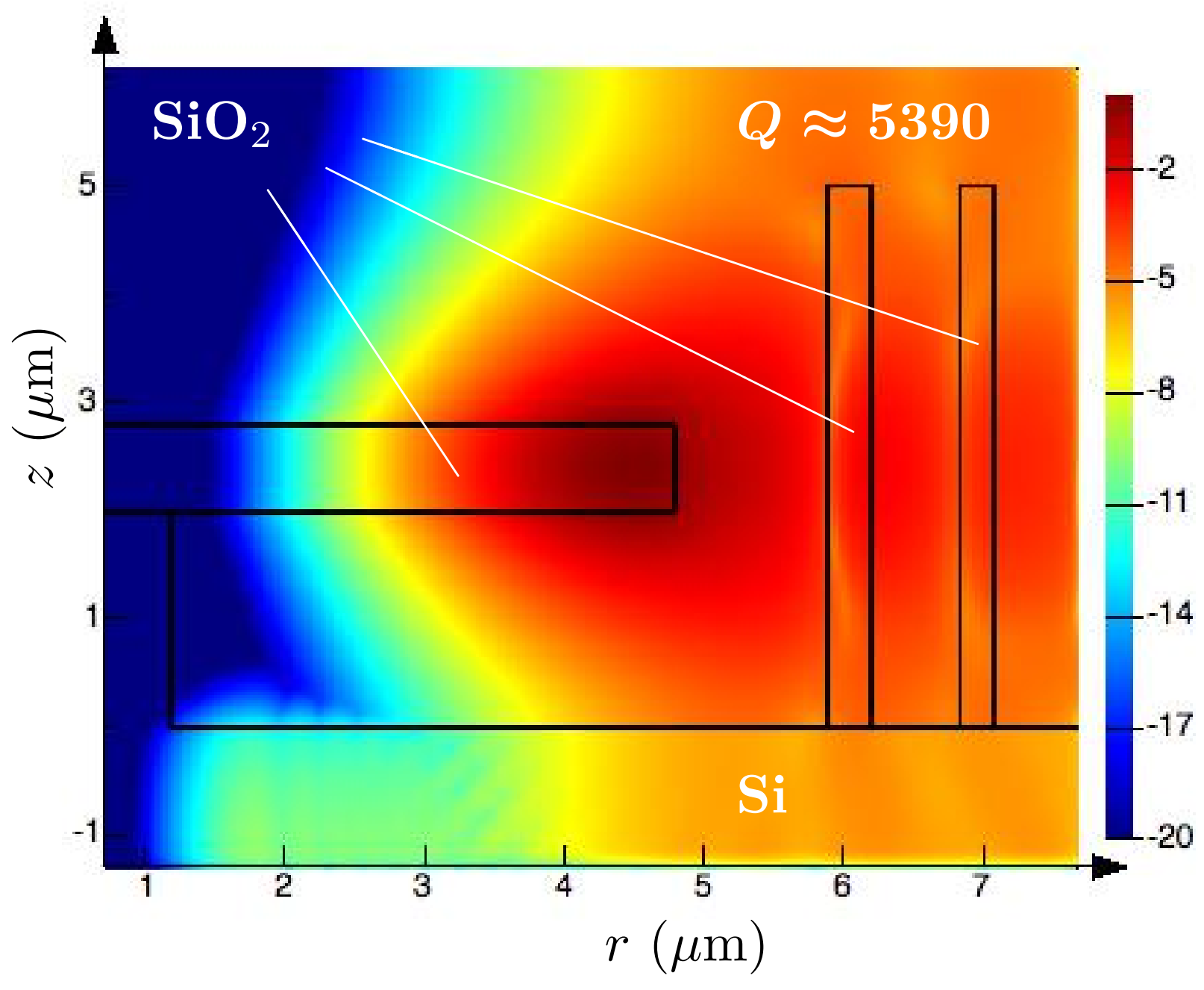
Exact solutions of Maxwells equation in 2D without the shield (left), with a shield consisting of a single ring (middle) and with 5 rings (right).

Realistic simulations in 3D

While the theory is developed in 2D, its conclusions are valid in 3D. We demonstrate this with a micro-disk on a pilar and a ridge cavity. In the latter case, the cavity can be excited more easily, e.g. by a buried waveguide [2].

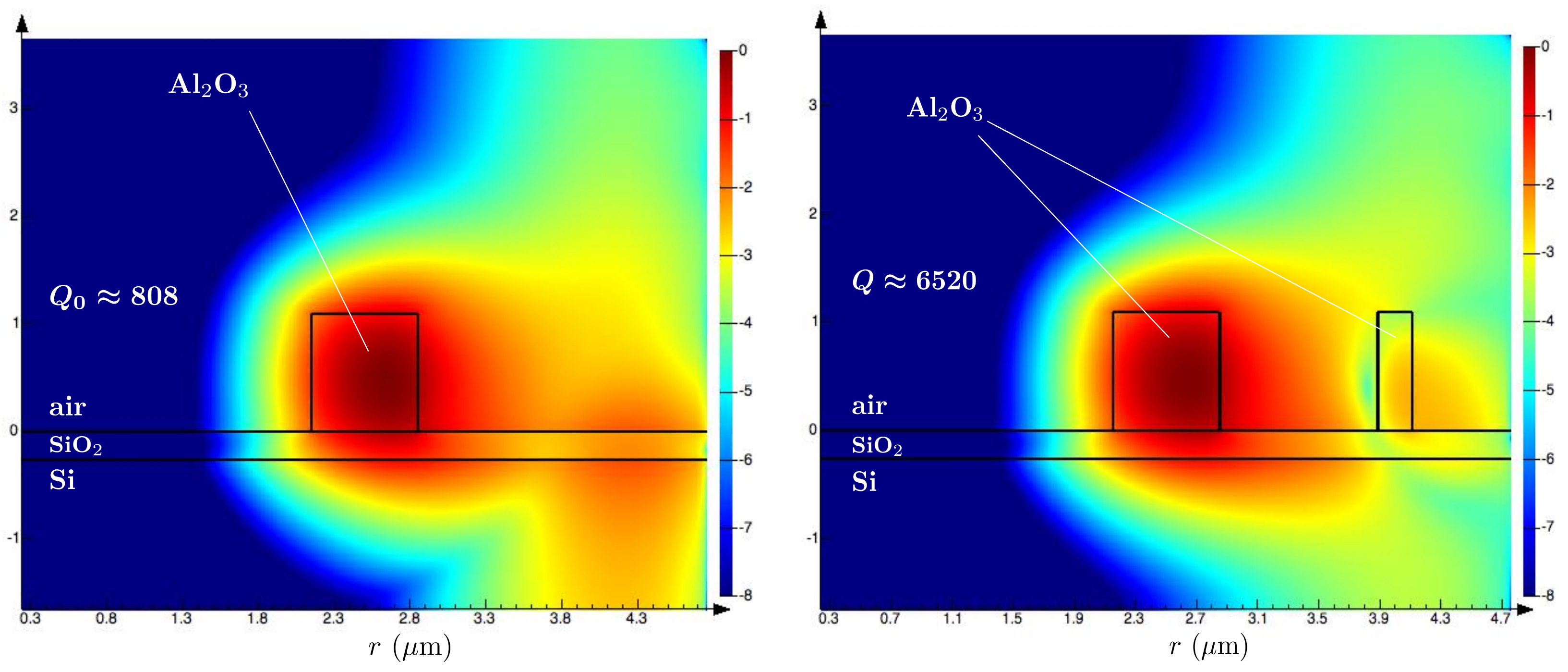






COMSOL simulations with a SiO$_2$ cavity ($n = 1.44$) on a Si base. $\lambda = 1.27\mu$m. Cavity radius: 4.78μm. Cavity thickness: 0.8μm. First shield inner radius and thickness: $r_a = 5.87\mu$m, $d_a = 0.33\mu$m. Second shield: $r_b = 6.84\mu$m, $d_b = 0.23\mu$m. Height of both shields: 5μm.

Below are simulation of a 2.8μm-radius Al$_2$O$_3$ ring cavity ($n = 1.65$) made of a circular ridge waveguide. $\lambda = 1.26\mu$m.



Here, one should take care of confinement in the vertical direction, as well as the polar divergence of the radiated field when designing the shield. But the principle still works.

Conclusion

- We have demonstrated mathematically in 2D and numerically in 3D the validity of the multi-ring shield.
- The physical mechanism is a form of radial Bragg reflection. However, the physics is more complicated than standard Bragg reflection with plane waves.
- A simplified formula allows one to rapidly evaluate the enhancement of Q in the presence of the shield.
- The external rings are too thin to guide light, so the mode stays confined in the cavity.
- Orders of magnitude enhancement of Q can be achieved for small cavities.
- The proposed structure can be manufactured with existing technologies and PIC platforms.
- Using this radiation shielding structure, the Purcell factor can be increased by orders of magnitudes. This may significant push the limit of WGM resonators in all their field of application, from sensing to lasing and quantum optics.

References

[1] Nirmalendu Acharyya and Gregory Kozyreff. Suppressing radiation from Whispering Gallery Modes. *arXiv e-prints*, page arXiv:1901.05740, Jan 2019.

[2] Zhan Su, Nanxi Li, Henry C. Frankis, E. Salih Magden, Thomas N. Adam, Gerald Leake, Douglas Coolbaugh, Jonathan D. B. Bradley, and Michael R. Watts. High-Q-factor Al$_2$O$_3$ micro-trench cavities integrated with silicon nitride waveguides on silicon. *Opt. Express*, 26(9):11161, apr 2018.

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